Lateral Buckling in Curtain Wall Systems

By Charles D. Clift,[1] Member, ASCE, and Walter J. Austin,[2] Fellow, ASCE

Abstract: Recent demand for increasingly complex exterior facades of commercial buildings has driven a need for more efficient high-performance curtain wall designs. Current codes can be improved by formulations that more closely parallel required criteria. In this paper, formulas for the elastic lateral buckling strength of slender monosymmetrical beams that represent curtain wall mullions with practical loading and supporting conditions have been derived by approximate energy procedures for four different assumptions of possible glazing restraint. The equations have been constructed in a general manner so that a large variety of glazing (bracing) positions is accommodated. Analysis of a typical design situation indicates that the currently used allowable compressive stresses of the Aluminum Association Specifications for buildings are extremely conservative for this application. Also, a method to experimentally verify the assumed glazing restraint is presented via the Southwell plot method. Use of the more accurate analyses herein will result in significant design efficiencies on many curtain wall systems.

Introduction

Glass curtain walls consist of large glass panes supported by vertical mullions and horizontal members between mullions. Wind forces on the curtain walls are transmitted to the building frame at the floor levels by the vertical mullions. The glazing transmits the wind forces to the mullions and also acts to restrain lateral-torsional motion of the mullions. The horizontal members between mullions also may restrain lateral-torsional buckling of the mullions. This paper is concerned with the lateral buckling strength of the long slender aluminum mullions, and the design of such members. Since there is practically no literature on the design of curtain wall mullions, most designers make very conservative assumptions.

Approximate analyses are given herein for several different supporting conditions that approximate minimum and maximum bracing and a realistic approximation to the restraint afforded by the glazing. For a typical application, comparisons are made of the allowable compressive stress given by current design procedures with corresponding values derived from the different theoretical analyses. These comparisons show the great influence of the restraint that may be offered by the glazing. Large increases in the allowable compressive stresses are possible. However, further study including experimental verification is suggested.

The cross section of a typical curtain wall mullion system is shown in Fig. 1. The mullion is represented in this paper by a prismatic monosymmetrical cross section, as in Fig. 2. The horizontal principal axis is denoted as x, the vertical principal axis as y and the longitudinal centroidal axis as z, forming

[1]Prin., Curtain Wall Design and Consulting, Inc., 10450 Brockwood Road, Dallas, TX 75238.

[2]Prof. Emeritus, Civ. Engrg. Dept., Rice Univ., P.O. Box 1892, Houston, TX 77251.

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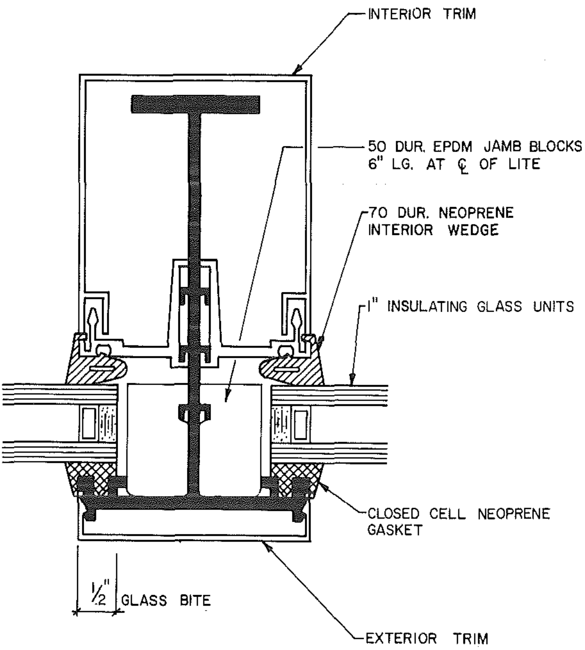

FIG. 1. Typical Curtain Wall Mullion

a right-hand system. The shear center has coordinates $(0, y_o)$. The horizontal displacement of the shear center is denoted as u and the rotation of the cross section as β. The uniform load w, positive in the positive y direction, is considered to be applied by the glazing a distance, $\bar{a}$, below the shear center, as shown in Fig. 2. The mullions span two equal floor heights, λ, and are

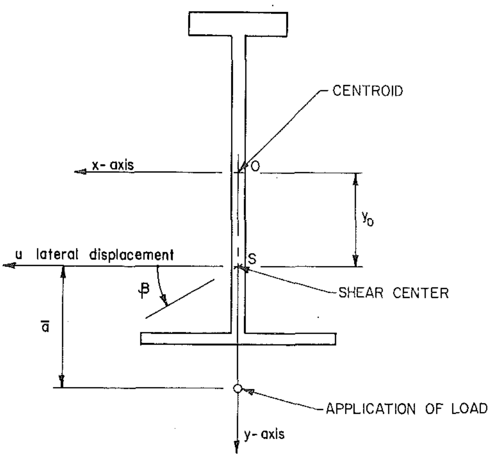

FIG. 2. Nomenclature of Mullion Cross Section and Axis System

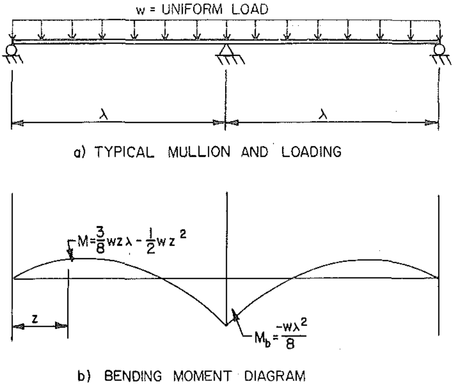

FIG. 3. Beam Analysis Model for Curtain Wall Mullion

considered as simply supported at the ends. Lateral wind loads are assumed to be static, uniformly distributed loads, as in Fig. 3(*a*). The corresponding bending moment about a horizontal axis is denoted as M. The bending moment diagram is shown in Fig. 3(*b*). Note that the wind pressure may be positive or negative on the glazing plane.

The current state of knowledge of lateral buckling of beams is described in Galambos (1988); this extensive presentation contains a very complete list of references. Texts by Timoshenko and Gere (1961), Bleich (1952), and Galambos (1968) give derivations of basic relationships and many elastic solutions, mostly of rectangular or I-shaped cross sections. Anderson and Trahair (1972) give the first extensive presentation of data on the theoretical elastic lateral buckling strength of simply supported and cantilever monosymmetrical beams subjected to uniform and concentrated loads. An experimental study on cantilevers verifies the validity of the theory.

Case I. Minimum Bracing Restraint

Assume first that the mullion of Fig. 2 is not restrained at all by the glass, but is freestanding. Each span of the two span beam will have a buckling mode as for a simply supported beam subjected to the loading and corresponding moment diagram of Fig. 3. A Rayleigh-Ritz approximate procedure is used.

The general potential energy expression for lateral buckling, derived by Masur and Milbradt (1957) is as follows:

$$V = \frac{1}{2}\int_{o}^{\lambda} [EI_y(u'')^2 + EC_w(\beta'')^2 + GJ(\beta')^2$$

$$+ 2M\beta u'' - 2K_oM(\beta')^2 + w\bar{a}\beta^2]dz \qquad (1a)$$

in which E = Young's modulus of elasticity; G = shear modulus; J = uniform torsion constant; C_w = warping torsion constant; I_y = moment of inertia about the y axis, each prime denoting one derivative with respect to z; and K_o = section constant given by the following expression:

$$K_o = y_o - \frac{1}{2I_x}\int_A y(x^2 + y^2)dA \qquad (1b)$$

in which A = area of the cross section.

For this case of a beam subjected to no lateral forces and with both ends "simply supported," the lateral curvature in Eq. 1a can be eliminated by the following relation, $u'' = -\beta M/EI_y$, to give the following potential energy expression

$$V = \frac{1}{2}\int_o^\lambda \left[EC_w(\beta'')^2 + GJ(\beta')^2 - \frac{M^2}{EI_y}\beta^2 - 2K_oM(\beta')^2 + w\bar{a}\beta^2\right]dz \qquad (2)$$

Assume a one-term approximation to the buckling shape

$$\beta = B\sin\frac{\pi z}{\lambda} \qquad (3)$$

in which B = magnitude coordinate. Minimizing the potential energy with respect to a variation in B, $\partial V/\partial B = 0$, gives Eq. 4.

$$(w\lambda^2)_{cr} = \frac{166}{\lambda}\sqrt{EI_yGJ}\left[\pm\sqrt{\delta_1^2 + 0.1191\left(1 + \frac{\pi^2}{\lambda^2}\frac{EC_w}{GJ}\right)} + \delta_1\right] \qquad (4)$$

in which

$$\delta_1 = \left(\frac{\bar{a}}{\lambda} + 0.08877\frac{K_o}{\lambda}\right)\sqrt{\frac{EI_y}{GJ}} \qquad (5)$$

The plus sign in front of the second radical in Eq. 4 indicates that the direction of loading coincides with the positive direction of the y axis; the minus sign indicates the opposite direction of loading. Note that both K_o and $\bar{a}$ may be negative; hence δ_1 may be negative.

Eq. 4 gives an upper bound. To check the accuracy of Eq. 4, the solution for a symmetric section, $K_o = 0$, with loading applied at the shear center, $\bar{a} = 0$, and with a wide range of properties was found by finite differences with a large number of divisions so that the answers are accurate. For this case Eq. 4 reduces to the following:

$$(w\lambda^2)_{cr} = 18.2\frac{\pi}{\lambda}\sqrt{EIGJ}\sqrt{1 + \frac{\pi^2}{\lambda^2}\frac{EC_w}{GJ}}$$

It was found that the numerical coefficient 18.2 is not constant but varies slightly with the parameter $GJ\lambda^2/EC_w$. The error varies from about 0.5 percent for very small values of the parameter to 2.5 percent for $(GJ\lambda^2/EC_w)$ = 1,000, a large value. Hence, it appears that Eq. 4 is sufficiently accurate for design purposes. In general, as noted by Galambos (1968) on p. 106, the Rayleigh-Ritz procedure applied to lateral buckling problems gives good results with relatively simple calculations, provided the assumed trial mode is well chosen.

Case II. Midstory Bracing

One may wish to consider the restraint afforded by intermediate wall elements in addition to the supports at the floor levels. Thus, assume the mullion is supported at midstory so that it cannot deflect laterally. Neglecting glazing restraints, it will then buckle into two half-waves in the story. One may obtain an approximate value for this problem by considering in Fig. 3 the mullion from outer support to midspan as "simply supported" for lateral-torsional buckling. This would be conservative. Thus, the potential energy expression is given by Eq. 2, except the limits of integration are from 0 to $\lambda/2$. Assume a trial buckled shape

$$\beta = B \sin \frac{2\pi z}{\lambda}$$

and minimize the potential energy. One obtains

$$(w\lambda^2)_{cr} = \frac{561}{\lambda} \sqrt{EI_y GJ} \left[\pm \sqrt{\delta_2^2 + 0.0352\left(1 + \frac{4\pi^2}{\lambda^2} \frac{EC_w}{GJ}\right)} + \delta_2 \right] \qquad (6)$$

in which

$$\delta_2 = \left(\frac{\bar{a}}{\lambda} - 0.903 \frac{K_o}{\lambda}\right) \sqrt{\frac{EI_y}{GJ}} \qquad (7)$$

Case III. Elastic Restraint

Next consider that the mullion is elastically restrained from both lateral deflection and rotation at the glazing plane that is a distance a below the shear center. Although a is usually equal to $\bar{a}$, there are ways to apply the load in which a is not equal to $\bar{a}$. Also, it may be noted that there are cases where the load is applied outside the flanges of the mullion. At the glazing plane, two restraining forces act: (1) The restraining torque per unit length $= K_\theta \beta$; and (2) the restraining lateral force resisting lateral deflection of the point of attachment of the glazing $= K_s \bar{u}$, in which $\bar{u}$ = lateral displacement of the point of attachment to the glazing. Hence, $\bar{u} = u - a\beta$.

The potential energy expression is given by Eq. 8

$$V = \frac{1}{2} \int_o^\lambda [EI_y(u'')^2 + EC_w(\beta'')^2 + GJ(\beta')^2 + K_\theta \beta^2 + K_s \bar{u}^2 + 2M\beta u''$$

$$- 2K_o M(\beta')^2 + w\bar{a}\beta^2] dz \qquad (8)$$

Eq. 8 is the general potential energy expression of Masur and Milbradt, Eq. 1*a*, supplemented by the strain energy of the rotational and lateral restraining springs $= \int_o^\lambda (K_\theta \beta^2 + K_s \bar{u}^2) dz/2$.

Assume

$$\beta = B \sin \frac{\pi z}{\lambda} \qquad (9a)$$

and

$$u'' = C\left[\sin \frac{\pi z}{\lambda} \left(\frac{3}{8}\frac{z}{\lambda} - \frac{1}{2}\left(\frac{z}{\lambda}\right)^2\right)\right] \qquad (9b)$$

and minimize with respect to B and C. The approximations of Eq. 9 were chosen so that when K_s and K_θ vanish, the solution will agree with that of Case I, Eq. 4. It isn't possible to arrange the answer in a simple explicit form. One obtains the following quadratic equation in $w\lambda^2$

$$0 = 0.003016 \frac{(w\lambda^2)^2}{EI_y} - w\lambda^2\left[\left(\frac{\bar{a}}{\lambda^2} + 0.08877 \frac{K_o}{\lambda^2}\right)C_1 - \frac{1}{106.9}\frac{K_s\lambda^4}{EI_y}\cdot\frac{a}{\lambda^2}\right]$$

$$- C_1\left(EC_w\frac{\pi^4}{\lambda^4} + GJ\frac{\pi^2}{\lambda^2} + K_\theta + K_s a^2\right) + \frac{1}{137.8}\frac{K_s^2\lambda^4 a^2}{EI_y} \quad \text{(10)}$$

in which

$$C_1 = 1 + \frac{1}{137.8}\frac{K_s\lambda^4}{EI_y} \quad \text{(11)}$$

For the case where the glazing is attached at the shear center, $a = 0$ (often about correct in practical cases), Eq. 10 gives the following solution

$$(w\lambda^2)_{cr} = \frac{166}{\lambda}\sqrt{EI_y GJ}\left[\pm\sqrt{(C_1\delta_1)^2 + 0.119C_1\left(1 + \frac{\pi^2 EC_w}{\lambda^2 GJ} + \frac{K_\theta\lambda^2}{\pi^2 GJ}\right)}\right.$$

$$\left. + C_1\delta_1\right] \quad \text{(12)}$$

Note δ_1 is given by Eq. 5.

The very simple trial deflection functions assumed in Eq. 9 are most appropriate for relatively weak springs, K_s, for which Eq. 10 should give good results. However, the critical load may be off considerably for strong lateral restraining springs.

There is also the possibility that the mullion will buckle into several waves at a lower load. To investigate this possibility the mullion is assumed to be subjected to a uniform moment given by Eq. 13

$$M = \frac{9}{128} w\lambda^2 \quad \text{(13)}$$

The following mode shape is assumed with n half-waves in each span

$$\beta = B \sin\frac{n\pi z}{\lambda} \quad \text{(14a)}$$

and

$$u = C \sin\frac{n\pi z}{\lambda} \quad \text{(14b)}$$

The following critical load is obtained:

$$(w\lambda^2)_{cr} = \frac{128}{9}\frac{n\pi}{\lambda}\sqrt{EI_y GJ}\left\{\pm\left[n^2\pi^2\delta_3^2 + C_2\left(1 + \frac{n^2\pi^2}{\lambda^2}\frac{EC_w}{GJ} + \frac{K_\theta\lambda^2}{n^2\pi^2 GJ}\right.\right.\right.$$

$$\left.\left.\left. + \frac{K_s a^2\lambda^2}{n^2\pi^2 GJ}\right) - \left(\frac{K_s\lambda^4}{n^4\pi^4 EI_y}\right)\left(\frac{K_s a^2\lambda^2}{n^2\pi^2 GJ}\right)\right]^{0.50} - n\pi\delta_3\right\} \quad \text{(15a)}$$

in which

$$C_2 = 1 + \frac{K_s \lambda^4}{n^4 \pi^4 E I_y} \quad (15b)$$

and

$$\delta_3 = \left[\frac{K_o}{\lambda} C_2 + \left(\frac{K_s \lambda^4}{n^4 \pi^4 E I_y}\right)\frac{a}{\lambda}\right] \sqrt{\frac{EI_y}{GJ}} \quad (15c)$$

This simplifies considerably for the case where the glazing is at the shear center, $a = 0$.

Case IV. Maximum Bracing Restraint

Finally, consider that the mullion is completely restrained from lateral deflection at the glazing plane and is elastically restrained from rotation. If $K_s \rightarrow \infty$ in Eq. 10, the following equation for the critical load is obtained

$$\omega\lambda^2(1.289a - 0.08877K_o - \bar{a})$$
$$= \pi^2 GJ\left(1 + \pi^2 \frac{EC_w}{\lambda^2 GJ} + \frac{K_\theta \lambda^2}{\pi^2 GJ} + \frac{137.8}{\pi^2}\frac{EI_y a^2}{\lambda^2 GJ}\right) \quad (16)$$

However, this cannot be expected to be very accurate because, as explained previously, the trail deflection function for u is not appropriate for this case. Therefore, a separate solution was obtained as described below.

The potential energy expression for this case is given by Eq. 8 with $\bar{u} = 0$. Obviously

$$u = a\beta \quad (17)$$

Using a one-term approximation, $\beta = B \sin \pi z/\lambda$, and minimizing, as before, one obtains Eq. 18

$$w\lambda^2(0.9112a - \bar{a} - 0.08877K_o)$$
$$= \pi^2 GJ\left(1 + \frac{\pi^2 EI_y a^2}{GJ\lambda^2} + \frac{\pi^2 EC_w}{GJ\lambda^2} + \frac{K_\theta \lambda^2}{\pi^2 GJ}\right) \quad (18)$$

Note that this is of the same form as Eq. 16.

If K_θ were of significant magnitude there is the possibility that the mullion would buckle into several waves at the lowest critical load. However, as will be shown, the buckling strength for practical mullions is so high (even for $K_\theta = 0$), there is no need to consider this possibility further.

Roeder and Assadi (1982) give the exact solution for a doubly symmetric wide flange beam ($K_o = 0$) with the lower flange prevented from lateral displacement ($a = d/2$) by a thin membrane ($K_\theta = 0$) and subjected to uniform moment. They give

$$M_{cr} = \left[\frac{\pi^2}{d\lambda^2}\left(\frac{d^2}{4} EI_y + EC_w\right) + \frac{GJ}{d}\right] \quad (19)$$

Eq. 18 gives for this beam with $\bar{a} = 0$

$$(w\lambda^2)_{cr} = \frac{2\pi^2}{0.9112}\left[\frac{\pi^2}{d\lambda^2}\left(\frac{d^2}{4}EI_y + EC_w\right) + \frac{GJ}{d}\right] \quad (20)$$

The corresponding maximum positive moment is

$$(M+)_{cr} = \frac{9}{128}(w\lambda^2)_{cr} = 1.52\left[\frac{\pi^2}{d\lambda^2}\left(\frac{d^2}{4}EI_y + EC_w\right) + \frac{GJ}{d}\right] \quad (21)$$

Thus, the form of this solution is the same as the exact solution of Roeder and Assadi for uniform moment and the coefficient is substantially larger, as would be expected.

Design Comparisons

The monosymmetric beam of Fig. 4 represents a simplified mullion that spans two stories with a story height, $\lambda = 150$ in. (381 cm). The glazing is roughly at the position of the shear center and hence it is assumed that the wind load is applied at the shear center. The mullion is assumed to be an extrusion composed of 6063-T6 aluminum.

Aluminum Association Specifications (1986)

The beam is fully supported against lateral buckling over the center support. Therefore, by Table 3.3.29 the allowable stress at this location is assumed to be 15 ksi (103.4 MPa) in either tension or compression. The limitations on compressive stress due to lateral buckling are assumed to apply only to the interior portions of the mullion, for which the maximum moment is $0.0703w\lambda^2$. For this example only downward forces will be considered.

Table 3.3.29 gives the following formula for allowable compressive stress in beams, in ksi, for $L_b/r_y^* \geq 94$

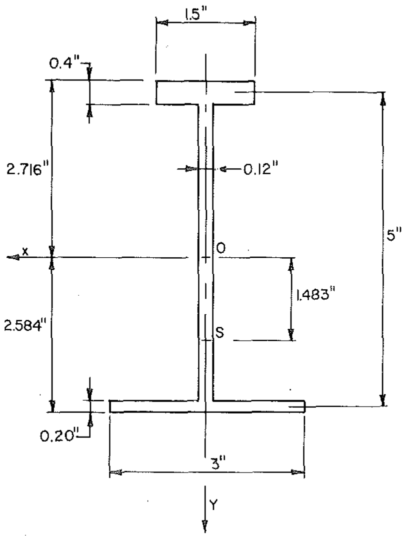

FIG. 4. Design Example of Mullion Cross Section Dimensions

$$F_b = \frac{87{,}000}{\left(\frac{L_b}{r_y^*}\right)^2} \quad (22a)$$

in which L_b = unsupported length of compression flange and r_y^* = radius of gyration about the y axis calculated as though both flanges were the same as the compressive flange. For the given mullion $L_b = \lambda = 150$ in. (381 cm); $r_y^* = 0.360$ in. (0.914 cm); and $(L_b/r_y^*) = 416$. For this very slender beam, Eq. 22*a* gives

$$F_b = 0.502 \text{ ksi } (3.46 \text{ MPa}) \quad (22b)$$

The Specifications for Aluminum Structures also give a more accurate formula for allowable compression stress in Section 4.9, in which r_y^* in Eq. 22 is replaced by an effective r_y^*, as follows for load applied to the bottom (tension) flange

$$\text{effective} \quad r_y^* = r_{ye}^* = \frac{k_b}{1.7}\sqrt{\frac{I_y^* d}{S^*}\left[0.50 + \sqrt{1.25 + 0.152\,\frac{J^*}{I_y^*}\left(\frac{L_b}{d}\right)^2}\right]} \quad (23)$$

in which k_b = coefficient = 1 for uniform transverse load as in this example; I_y^* = moment of inertia about axis parallel to web; S^* = section modulus; d = depth of section; and J^* = uniform torsion constant, all computed as though both flanges were the same as the compression flange. Thus, I_y^* = 0.226 in.^{4} (9.41 cm^4); S^* = 3.07 in.^{3} (50.3 cm^3); d = 5.30 in. (13.46 cm); and J^* = 0.0666 in.^{4} (2.77 cm^4). Then, by Eq. 23, r_{ye}^* = 0.944 in. (2.40 cm); and by Eq. 22, F_b = 3.45 ksi (23.8 MPa).

Thus, the allowable compressive stress by this provision is 6.9 times larger than that given by the provision of Section 3.4.11.

Case I. No Bracing Restraint

The critical value of $w\lambda^2$ is given by Eq. 4, in which for the cross section of Fig. 4, I_y = 0.563 in.^{4} (23.4 cm^4); $G = 0.385E$; E = 10,000 ksi (68,950 MPa); J = 0.0429 in.^{4} (1.79 cm^4); C_w = 2.250 in.^{6} (604 cm^6); and K_o = 1.450 in. (3.68 cm). One obtains $(w\lambda^2)_{cr}$ = 384 in.-kip (43.4 kN · m), and the corresponding maximum positive moment = $(M+)$ max = $0.0703w\lambda^2$ = 27.0 in.-kip (3.05 kN · m). The corresponding compressive stress at elastic buckling is $f_c = M/S$ = 8.58 ksi (59.2 MPa) in which S = 3.15 in.^{3} (51.6 cm^3). If one applies a factor of safety of 1.65 as is used for the Alluminum Association formulas for buildings, one obtains the following allowable compressive stress for the interior region

$$F_b = 5.20 \text{ ksi } (35.8 \text{ MPa}) \quad (24)$$

This allowable compressive stress is 10 times larger than the value given by Eq. 22 and 1.5 times larger than the value given using the effective r_{ye}^* of Eq. 23. The Case I analysis neglects all restraint offered by the glazing; the additional strength results partly from an accurate evaluation of the true bending moment variation in Eq. 4 and partly from the approximate nature of the Aluminum Association specification procedure when applied to this case.

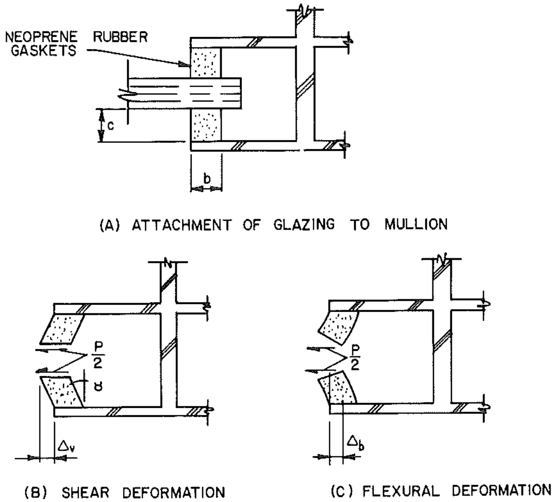

FIG. 5. Glazing Pocket Detail for Modeling Elastic Restraint

Case II. Midspan Bracing

If an intermediate element prevents lateral displacement at midstory of the given mullion, the critical load is approximated by Eq. 6, giving $(w\lambda^2)_{cr}$ = 591 in.-kip (66.8 kN · m). This gives a maximum positive moment = 41.6 in.-kip (4.70 kN · m), which corresponds to a critical compressive stress of 13.2 ksi (91.0 MPa). Again, assuming a factor of safety of 1.65, one obtains the following allowable stress

F_b = 8.00 ksi (55.2 MPa) . . . . (25)

Thus the intermediate supporting element increased the strength over the freestanding mullion, Case I, by a factor of 1.54.

Case III. Elastic Restraint

Small restraints can greatly increase the lateral-torsional buckling strength of the aluminum mullions. A reliable estimate of the restraint constants K_θ and K_s provided by the glazing is not available. As a very rough estimate of the minimum elastic restraint, assume that $K_\theta = 0$ and that K_s is provided by the elasticity of the soft Neoprene seals only as it seems reasonable to assume that the glass does not move horizontally in its plane.

The attachment of the glass to one side of the mullion is shown in Fig. 5(*a*). Although the glass is assumed fixed and the mullion moves, it is easier to visualize the mullion as fixed and the glass as moving, as in Figs. 5(*b*) and 5(*c*). The very complex stress-displacement state in the seals is roughly approximated herein as composed of pure shear and pure flexure. The total displacement of the glass Δ is the sum of the displacements due to shear, Δ_v, and flexure, Δ_b. The force per unit length of seal of each glass pane is denoted as P. Each seal resists one-half of this force.

First consider the displacement due to shear. Assume the load $P/2$ causes uniform shearing stress in the seal. Then

$$\text{shearing stress} = \tau = \frac{P}{2b} \quad \text{(26)}$$

in which b is the width of the seal, as in Fig. 5(a). The corresponding shearing strain is

$$\text{shearing strain} = \gamma = \frac{\tau}{G} = \frac{3}{2}\frac{P}{bE} \quad \text{(27)}$$

as it is assumed for the seal material that $G = E/3$. The displacement due to shear is then

$$\Delta_v = \gamma c = \frac{3}{2}\frac{P}{E}\frac{c}{b} \quad \text{(28)}$$

in which c = height of the seal, Fig. 5(a).

As a conservative assumption assume simple flexure of each seal as a cantilever beam. Then the beam displacement is

$$\Delta_b = \frac{\left(\frac{P}{2}\right)c^3}{3EI_s} = \frac{\left(\frac{P}{2}\right)c^3}{3E\left(\frac{b^3}{12}\right)} = \frac{2Pc^3}{Eb^3} \quad \text{(29)}$$

in which I_s = moment of inertia of seal per unit width.

The total displacement Δ is

$$\Delta = \Delta_v + \Delta_b = \frac{P}{E}\left(\frac{3}{2}\frac{c}{b} + 2\frac{c^3}{b^3}\right) \quad \text{(30)}$$

Assuming that a glass pane is restraining the mullion on each side, the total spring stiffness K_s is

$$K_s = \frac{2P}{\Delta} = \frac{2E}{\left(\frac{3}{2}\frac{c}{b} + 2\frac{c^3}{b^3}\right)} \quad \text{(31)}$$

Assume as typical dimensions $b = c = 0.250$ in. (0.635 cm) and for soft Neoprene rubber under short time loading, $E = 2{,}000$ psi (13.8 MPa). Then, $K_s = 1{,}143$ lb/in./in. (788 N/cm/cm). Obviously, this is a ballpark estimate but it should be low since consistently conservative assumptions have been made. In the following calculation $K_s = 1{,}000$ lb/in./in. (690 N/cm/cm) has been assumed.

Assume that the central plane of the glazing is at the shear center; hence, $a = \bar{a} = 0$. For a story height $\lambda = 150$ in. (381 cm); as before, the two-wave solution of Case III gives the lowest critical load as follows: $(w\lambda^2)_{cr} = 965$ in.-kip (109 kN · m). This is 2.51 times larger than the critical load of Case I. The elastic critical compressive stress is 21.6 ksi (149 MPa). The compressive yield strength of 6063-T6 aluminum is 25 ksi (172 MPa). Therefore, this case should fail inelastically at about 17.7 ksi (122 MPa).

Or using a factor of safety of 1.65 gives the following allowable stress

F_b = 10.7 ksi (74 MPa) . . . . (32)

This is about three times the value allowed by the Specifications for Aluminum Structures and over twice as much as the suggested allowable stress for the freestanding beam, Case I.

Note that in this example a minimum glazing restraint is assumed as well as a uniform moment. The true critical load is probably much higher.

Case IV. Maximum Bracing Restraint

This case assumes that the mullion is completely restrained from lateral deflection at the glazing plane and elastically restrained from rotation. For this example assume the glazing is at the shear center and neglect entirely the rotational restraint. Then $a = \bar{a} = 0$ and $K_\theta = 0$. Eq. 18 gives the following result: $(w\lambda^2)_{cr} = -13{,}420$ in.-kip ($-1{,}516$ kN · m). Thus, the absolute value of $w\lambda^2$ is 35 times larger than the value of Case I. The negative sign means that the wind force would have to be reversed (suction). This result shows that if the given mullion is prevented from lateral displacements by the glazing located at the shear center there is really no lateral buckling problem and the allowable stresses do not have to be reduced.

Comment

The dramatic difference in results between the regular Aluminum Association allowable compressive stress equation, its more precise effective r_y equation, and an allowable compressive stress based on the minimum possible critical load, Case I, is readily discerned. Case I gives an increased capacity of 51% compared to the effective radius equation and an increase of over 900% compared to the regular equation. The contrasts are even greater when comparisons to Cases II, III, and IV are made. In Case III an attempt is made to conservatively estimate the elastic restraint offered by the glazing. This solution gives an increase in the allowable stress over that of Case I of over 100%.

If the elastic restraint afforded by the glazing can be estimated reliably and conservatively, it is certain that higher allowable stresses could be used with great savings of materials. It seems desirable to experimentally evaluate the behavior of curtain wall construction.

Experimental Studies

The critical buckling strength of the very slender mullions used in curtain wall construction is very sensitive to the restraint offered by the glazing. As discussed, this restraint is difficult to estimate accurately. However, it is presently the industry standard to carry out full-scale structural mock-up testing on new or major curtain wall designs. This provides an excellent opportunity to collect data that can be used to establish the strength and behavior of the slender mullions.

The Southwell plot was originally proposed as a nondestructive procedure for extrapolating the elastic critical load of concentrically loaded struts from tests in which the load is increased to a fairly high value but not to failure.

The theory is unquestionably correct for the simple strut problem. The theory and application of the Southwell-type procedures for lateral buckling problems is described by Trahair (1969) and Attard (1983). The plot of the product wind pressure times measured displacement versus measured displacement should be a straight line with the slope equal to the critical wind pressure. The plot may not be a straight line with the correct slope until the wind pressure reaches a significant percent of the critical value and, of course, the stress-strain behavior must be linear. The measured displacement could be the sideways displacement of the compression flange at a point of maximum displacement. Assuming the mullions have some initial crookedness, if in a test one does not measure any progressively increasing lateral displacement as one increases the load up to the maximum, then the conclusion would be that the maximum is simply nowhere near the critical value. In the several tests witnessed by the senior writer, such has been the case, as the measured displacements have been negligible. The useful strength of the curtain walls in these tests depended on the yield strength of the mullions, with no lateral buckling tendencies observed.

Summary and Conclusions

Formulas for the elastic lateral buckling strength of slender monosymmetrical beams that represent curtain wall mullions with practical loading and supporting conditions have been derived by approximate energy procedures for four different assumptions of possible elastic restraint. These are: (1) No lateral bracing; (2) bracing at midstory only; (3) elastic restraint against lateral displacement and twisting displacement at the glazing level; and (4) maximum bracing that completely prevents lateral displacements at the glazing level. The beams span two stories and are subjected to uniform load.

A typical design situation is evaluated. The allowable compressive stresses obtained by applying the two Aluminum Association Specification (AAS) formulas for unbraced beams are shown to be very conservative. The solution based upon no lateral bracing leads to an allowable compressive stress 51% higher than given by AAS. The others are higher. The most realistic assumption, the third, with very conservative assumptions of glazing restraint leads to an allowable compressive stress about three times the AAS value.

Experimental studies performed to verify by test practical designs by the senior author show no progressive displacement failures and suggest there is sufficient bracing restraint by the glazing to prevent lateral buckling failure.

With additional study it should be possible to establish realistic design codes that will allow far more economical designs than are now possible.

Appendix I. References

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = area of cross section of mullion;
a = distance of elastic restraint below shear center;
$\bar{a}$ = distance of point of load application below shear center;
B, C = constants in series expressions;
b, c = dimensions of seal;
C_w = warping-resistance constant;
d = depth of wide-flange or I section;
E = Young's modulus of elasticity;
F_b = allowable design compressive stress;
f_c = maximum compressive stress at elastic buckling;
G = shearing modulus;
I_x, I_y = moments of inertia about x and y axes, respectively;
J = uniform torsion cross-section constant;
K_o = cross-section constant;
K_θ, K_s = stiffness constants for rotational and transverse spring supports, respectively;
k_b = load coefficient;
L_b = unsupported length of compression flange;
M = bending moment about x axis;
n = number of half-waves of buckling displacement;
P = force per unit length of seal;
r_y = radius of gyration about y axis of wide flange beam;
r_{ye} = effective r_y;
S = section modulus for maximum compressive stress for bending about x axis;
u = horizontal displacement of shear center, a function of z;
V = potential energy of slightly displaced system;
w = uniform load;
x, y = principal axes of inertia and Cartesian coordinates;
y_o = y coordinate of shear center;
z = longitudinal coordinate;
β = rotation of cross section about shear center, a function of z;
γ = shearing strain (τ/G);

$\Delta, \Delta_b, \Delta_c$ = displacements of seal;
$\delta_1, \delta_2, \delta_3$ = dimensionless parameters;
λ = story height;
τ = shearing strain in seal;
$(\ )^*$ = quantity computed for I beam with unequal flanges as if both flanges were the same as the compression flange, holding the total depth constant; and
$(\ )_{cr}$ = elastic critical buckling value.